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Thermal Instability of Hydromagnetic Couple-Stress Fluid : Effect of Suspended Particles

Abstract

In this paper, the thermal instability of couple stress fluid layer heated from below in the presence of magnetic field has been discussed. Necessary condition for instability and sufficient condition for stability have been obtained for oscillatory as well as for non-oscillatory modes.

Keywords: Thermal instability, Couple-Stress Fluid, Rotation and Magnetic Field.

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Introduction

Instability of compressible or incompressible flows has been studied extensively by a number of research workers in past few decades. In almost all such investigations, the Boussinesq's approximation is used to simplify the equations of motions. **Plainswamy** and **Puroshotham**[1] have examined the stability of shear flow of stratified fluid with fine dust and the effect of fine dust to increase the region of instability. The effect of particle mass and heat capacity on the onset of Benard convection has been considered by **Scanlon** and **Segel**[2]**.** A layer of Rivlin-Ericksen elastico-viscous, compressible fluid heated and soluted from below in the presence of suspended particles (fine dust) is considered by **Sharma** and **Sharma**[3]. **Sharma and Rani**[4] examined the study of double-diffusive convection with fine dust (suspended particles). **Sharma and Sharma**[5] have examined the effect of suspended particles on couple-stress fluid heated from below in the presence of rotation and magnetic field. The theoretical and experimental study of the onset of Benard convection in Newtonian fluids, under varying assumptions of hydrodynamics, has been given by **Chandrasekhar**[6]**.** The fluid has been considered to be Newtonian throughout the study. **Sharma and Kumar**[7] examined the effect of suspended particles on the thermal instability of Rivlin-Ericksen elastico-viscous fluid. **Sharma and Sharma**[8] have examined the couple-stress fluid heated from below in porous medium. The thermal convection in couple-stress fluid in porous medium in hydromagnetics is considered by **Sharma and thakur**[9].**Rana Goel and Agrawal**[10] have studied the study of Rivlin-Ericksen Elastico viscous fluid heated and soluted from below in the presence of suspended particle with the effect of compressibility. With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable.**Stokes**[11] proposed and postulated the theory of couple-stress fluid. In almost all such investigations, the Boussinesq's approximation is used to simplify the equations of motion. **Jeffrey**[12] tried to provide a justification of the Boussinesq's approximation for steady motion of the fluids. In the present analysis, we have examined, within the framework of linear analysis, the thermal instability of couple-stress fluid heated from below in the presence of rotation and magnetic field with the effect of suspended particles.

Formulation of the Problem

Consider an infinite, horizontal, incompressible couple-stress fluid layer of thickness d, heated from below so that, the temperature and the density at the bottom surface z = 0 are T_0 and ρ_0 respectively and at the upper surface z = d are T_d and ρ_d. Also the temperature gradient $\beta = \left|\frac{dT}{dz}\right|$ is considered.

The equations of couple-stress fluid, equations of motion and continuity for the particles and equation of heat conduction are:

$$\frac{\partial \bar{q}}{\partial t} + (\bar{q}\cdot\nabla)\bar{q} = -\frac{1}{\rho_0}\nabla p + \bar{g}\left(1+\frac{\delta p}{\rho_0}\right) + \left(v - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\bar{q} + \frac{KN}{\rho_0}(q_d - q) \qquad \ldots(1)$$

$$\nabla\cdot\bar{q} = 0 \qquad \ldots(2)$$

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$$mN\left[\frac{\partial\bar{q}_d}{\partial t}+\left(\bar{q}_d\cdot\nabla\right)\bar{q}_d\right]=KN\left[\bar{q}-\bar{q}_d\right] \quad ...(3)$$

$$\frac{\partial N}{\partial t}+\nabla\cdot\left(N\bar{q}_d\right)=0, \quad ...(4)$$

$$\frac{\partial T}{\partial t}+\left(\bar{q}\cdot\nabla\right)T+\frac{mNc_{pt}}{\rho_0 c_v}\left(\frac{\partial}{\partial t}+\bar{q}_d\cdot\nabla\right)T=\kappa\nabla^2 T \quad ...(5)$$

where

$\bar{g}$ = (0, 0, -g), acceleration due to gravity,
mN = mass of the particle per unit volume,
c_v = heat capacity of the fluid,
c_{pt} = heat capacity of the particle
μ' = couple- stress viscosity
K = thermal diffusivity,
T = temperature,

The equation of basic state is given by

$$\rho=\rho_0\left[1-\alpha\left(T-T_0\right)\right] \quad ...(6)$$

where suffix zero indicates the reference state. Further, the basic solution is

$$\left.\begin{array}{ll}\bar{q}=(0,0,0), & \bar{q}_d=(0,0,0),\\ T=T_0-\beta z, & \rho=\rho_0\left(1+\alpha\beta z\right)\end{array}\right\} \quad ...(7)$$

and $N=N_0$, a constant

Perturbations and Normal Mode Analysis

Now we suppose that the solution in the basic state is slightly perturbed so that every physical quantity is assumed to be the sum of a mean and a fluctuating component, later designated as primed quantity and assumed to be very small in comparison to its basic state value. The small disturbances are assumed to be the functions of the space as well as time variables.

Let $\delta\rho$,N, $\delta p,\theta,$ q(u,v,w) and q_d (l,r,s) denote the perturbations in density, suspended particles, number density N_0 , pressure p, temperature T, couple stress fluid velocity (0,0,0) and particle velocity (0,0,0) respectively. Thus, the perturbation θ in temperature is given by

$$\delta\rho = -\alpha\rho_0\theta \quad ...(8)$$

The perturbations are analysed in terms of the normal modes, we assume that the perturbations quantities are of the form

$$\left[w,\theta\right]=\left[W(z),\theta(z)\right]\exp\left(ik_x x+ik_y y+nt\right) \quad ...(9)$$

where k_x and k_y are wave number in x and y directions respectively and $k=\sqrt{k_x^2+k_y^2}$ is the resultant wave number and n is, in general, a complex constant.

For the considered form of perturbations given in equation (9), linearized equations become

$$\frac{\partial\bar{q}}{\partial t}=-\frac{1}{\rho_0}\nabla\delta p-\bar{g}\alpha\theta+\left(v-\frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\bar{q}+\frac{KN_0}{\rho_0}\left(\bar{q}_d-\bar{q}\right), \quad ...(10)$$

$$\nabla\cdot\bar{q}=0, \quad ...(11)$$

$$mN_0\frac{\partial\bar{q}_d}{\partial t}=KN_0\left(\bar{q}-\bar{q}_d\right), \quad ...(12)$$

$$(1+h)\frac{\partial\theta}{\partial t}=\beta\left(w+hs\right)+\kappa\nabla^2\theta, \quad ...(13)$$

where $\kappa=\dfrac{q}{\rho_0 c_v}$

and $h=\dfrac{mN_0 c_{pt}}{\rho_0 c_v}$,

On eliminating various physical quantities from these equations, we get the final stability equation as

$$\left(D^2-a^2\right)\left[\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]W=-\frac{g\alpha d^2a^2\theta}{v} \quad ...(14)$$

$$\left(D^2-a^2-Hp_1\sigma\right)\theta=-\frac{\beta d^2}{\kappa}\frac{\left(H+\tau_1\sigma\right)}{\left(1+\tau_1\sigma\right)}W \quad ...(15)$$

where $a=kd$, $\sigma=\dfrac{nd^2}{v}$, $\tau=\dfrac{m}{\kappa}$,

$$\tau_1=\frac{\tau v}{d^2},\ M=\frac{mN_0}{\rho_0},\ p_1=\frac{v}{\kappa},$$

$$H=1+h,\ F=\frac{\mu'}{\rho_0 d^2 v} \text{ and } D=\frac{d}{dz}.$$

The boundary conditions for the equations (14) and (15) are

$$W=0,\theta=0 \text{ at } z=0 \text{ and } z=1 \quad ...(16)$$

These boundary conditions in non-dimensional form become

$$D^2W=D^4W=0 \text{ at } z=0 \text{ and } z=1. \quad ...(17)$$

Now, multiplying equation (14) by W* (complex conjugate of W) using equation (15) with the boundary conditions (15) and (16) we obtain the stability equation

$$\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right)I_1+I_2+FI_3=\frac{g\alpha a^2\kappa}{v\beta}\left(\frac{1+\tau_1\sigma^*}{H+\tau_1\sigma^*}\right)\left(I_4+Hp_1\sigma^*I_5\right), \quad ...(18)$$

where

$$I_1=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz,$$

$$I_2=\int_0^1\left(|D^2W|^2+2a^2|DW|^2+a^4|W|\right)dz,$$

$$I_3=\int_0^1\left(|D^3W|^2+3a^2|D^2W|^2+3a^4|DW|^2+a^6|W|^2\right)dz,$$

$$I_4=\int_0^1\left(|D\theta|^2+a^2|\theta|^2\right)dz,$$

and $I_5=\int_0^1|\theta|^2dz$,

The integrals I_1-I_5 are all positive definite.

On substituting $\sigma=i\sigma_i$ in equation (18), where σ_i is real, we get

$$i\sigma_i\left(1+\frac{M}{1+i\tau_1\sigma_i}\right)I_1+I_2+FI_3=\frac{g\alpha a^2\kappa}{v\beta}\left(\frac{1-i\tau_1\sigma_i}{H-i\tau_1\sigma_i}\right)\left(I_4-iHp_1\sigma_iI_5\right)$$

or

$$i\sigma_i\left[1+\frac{M}{1+\tau_1^2\sigma_i^2}-i\frac{M\tau_1\sigma_i}{1+\tau_1^2\sigma_i^2}\right]I_1+\left(I_2+FI_3\right)$$
$$=\frac{g\alpha a^2\kappa}{v\beta}\left[\frac{H+\tau_1^2\sigma_i^2}{H^2+\tau_1^2\sigma_i^2}+i\frac{\tau_1\sigma_i(1-H)}{H^2+\tau_1^2\sigma_i^2}\right]\left(I_4-iHp_1\sigma_iI_5\right) \quad .,.(19)$$

Thus, in other words means that the modes are taken to be neutral.

Equating the imaginary part of equation (19) from both sides, we get

$$\sigma_i\left[\left(1+\frac{M}{1+\tau_1^2\sigma_i^2}\right)I_1-\frac{g\alpha a^2\kappa}{v\beta}\left\{\frac{\tau_1(1-H)}{H^2+\tau_1^2\sigma_i^2}I_4-\frac{H+\tau_1^2\sigma_i^2}{H^2+\tau_1^2\sigma_i^2}.Hp_1I_5\right\}\right]=0 \quad ...(20)$$

If $\sigma_i\neq 0$, then equation (20) provides

$$\left(1+\frac{M}{1+\tau_1^2\sigma_i^2}\right)I_1-\frac{g\alpha a^2\kappa}{v\beta}\left\{\frac{\tau_1(1-H)}{H^2+\tau_1^2\sigma_i^2}I_4-\frac{H+\tau_1^2\sigma_i^2}{H^2+\tau_1^2\sigma_i^2}.Hp_1I_5\right\}=0$$

or $$\left(1+\frac{M}{1+\tau_1^2x}\right)I_1-A\left\{\frac{\tau_1(1-H)}{H^2+\tau_1^2x}I_4-\frac{H+\tau_1^2\sigma_i^2}{H^2+\tau_1^2x}.Hp_1I_5\right\}=0$$

where $A=\dfrac{g\alpha a^2\kappa}{v\beta}$ …(21)

and $x=\sigma_i^2$,

On simplification, equation (21), we get

$$Px^2+Qx+R=0 \quad \text{…(22)}$$

where

$$P=\left(I_1+\frac{g\alpha a^2\kappa}{v\beta}Hp_1I_5\right)\tau_1^4$$

$$Q=\left(1+M+H^2\right)\tau_1^2I_1+\frac{g\alpha a^2\kappa}{v\beta}\left[(H-1)\tau_1+I_4+(1+H)Hp_1I_5\right]\tau_1^2$$

and

$$R=(1+M)H^2I_1+\frac{g\alpha a^2\kappa}{v\beta}\left\{(H-1)\tau_1I_4+H^2p_1I_5\right\}$$

Analytical Discussion

In this section, we shall prove some important theorems with the help of equation (22).

Clearly, P,Q and R all are positive under the condition that B is positive. In that case equation (22) has no positive roots and there are either negative or complex roots which is impossible because σ_i is real, so that $\sigma_i^2\,(=x)$ is positive.

Now, we prove the following theorems :

Theorem 1

If the coefficients P, Q and R in equation (22) are either all positive or all negative then the neutral modes do not exist.

Proof

Let the modes be neutral, so that $\sigma_r=0$. Then if all the coefficients P, Q and R are either positive or negative then equation (22) do not admit any positive root and the roots will be either all negative or one negative and one complex. In either case, it leads to the contradiction to the fact that x being equal to $\sigma_i^2\,(\sigma_i \text{ is real})$ is positive definite.

This contradiction is due to the fact that the existence of neutral modes has been assumed. Therefore, under these conditions of the theorem, neutral modes do not exist.

Theorem 2

The neutral modes are oscillatory under the condition $R\neq 0$.

Proof : It $R\neq 0$, then equation (22) implies that $\sigma_i\neq 0$. It means that the neutral modes are oscillatory.

Remark : That all the three coefficients P, Q and R are either positive or negative is possible in many physical situation. For example, it β is positive then all the three coefficients P, Q and R are positive definite. The above discussion leads to the fact that neutral modes do not exist under the condition $\beta>0$.

Likewise it is also possible to have all the three coefficients as negative. For example if the condition $\beta<0$ and

$$I_1>\frac{g\alpha a^2\kappa}{\nu|\beta|}.Hp_1I_5,$$

$$\left(1+M+H^2\right)I_1>\frac{g\alpha a^2\kappa}{\nu|\beta|}\left[(H-1)-\tau_1I_4+(1+H)Hp_1I_5\right]$$

and $$(1+M)H^2I_1>\frac{g\alpha a^2\kappa}{\nu|\beta|}\left[(H-1)\tau_1I_4+H^2p_1I_5\right]$$

Then P, Q and R are all negative.

Now, if non-oscillatory modes $(\sigma_i=0)$ exist then putting $\sigma=\sigma_r$ in equation (18),

we get

$$\sigma_r\left[1+\frac{M}{1+\tau_1\sigma_r}\right]I_1+I_2+FI_3$$
$$-\frac{g\alpha a^2\kappa}{\nu\beta}\left(\frac{1+\tau_1\sigma_r}{H+\tau_1\sigma_r}\right)\left(I_4+Hp_1\sigma_rI_5\right)=0$$

or $$P'\sigma_r^3+Q'\sigma_r^2+R'\sigma_r+S'=0 \quad \text{…(23)}$$

where $$P'=\left(I_1-\frac{g\alpha a^2\kappa}{\nu\beta}\cdot Hp_1I_5\right)\tau_1^2$$

$$Q'=\left\{(1+H+M)I_1+(I_2+FI_3)\right\}\tau_1-\frac{g\alpha a^2\kappa}{\nu\beta}\left(\tau_1I_4+2Hp_1I_5\right)$$

$$R'=H(1+M)I_1+(I_2+FI_3)(1+H)\tau_1-\frac{g\alpha a^2\kappa}{\nu\beta}\left(Hp_1I_5+2\tau_1I_4\right)$$

and $$S'=(I_2+FI_3)H-\frac{g\alpha a^2\kappa}{\nu\beta}I_4$$

We now prove the following theorems:

Theorem 3

If P' and S' are of opposite signs, then non-oscillatory modes are stable.

Proof

Let P' and S' be of opposite signs. Then it follows from equation (23) that the product of the roots is negative, which ensures that at lest one root is negative. Infact one root is negative, then the remaining two roots can be all positive or complex.

Therefore, the existence of positive and negative values of σ_r is ensured under the condition that P' and S' are of opposite signs. In another words, non-oscillatory modes $(\sigma_i=0)$ are stable $(\sigma_r<0)$ under the condition that P' and S' are of opposite signs. It is to be noted that this result does not at all depend upon the other coefficients Q' and R'. It is to be noted that the coefficients P' and S' are of same signs it $\beta<0$. Therefore this theorem is not valid under this condition of β.

Theorem 4

It P', Q', R' and S' are all positive, then the system is stable.

Proof

It is to be observed that all the coefficients are positive definite under the condition $\beta<0$. It means that the equation (23) does not allow any positive value of σ_r implying, thereby, the stability of the system. Hence the system is stable.

Conclusion

The paper concerns the linear stability analysis of thermosolutal Hydromagnetic couple-stress fluid in the presence of suspended particles. Important results obtained in this paper include different conditions of stability, existence of oscillatory modes, non-oscillatory modes, discussion for stable and unstable modes, if exist in the problem. Finally we have taken the parturbations in terms of the normal mode analysis and then we have examined the thermal linear stability analysis of couple- stress fluid heated from below in the presence of rotation and magnetic field with the effect of suspended particles.

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